[TOPIC 1] Euclid's Division Lemma and Fundamental Theorem of Arithmetic

Summary

Euclid's Division Lemma

Dividend = divisor × quotient + remainder.

Given two positive integers a and b. There exist unique integers q and r satisfying

$a = bq + r$ where $0 \le r < b$

where a is dividend, b is divisor, q is quotient and r is remainder.

- ***If $a = bq + r$, then every common divisor of a and b is a common divisor of b and r also.***

Euclid's Division Algorithm

To obtain the HCF of two positive integers, say c and d, with $c > d$, follow the steps below:

Step 1: Apply Euclid's division lemma, to c and d. So, we find whole numbers, q and r such that $c = dq + r$, $0 \le r < d$.

Step 2: If $r = 0$, d is the HCF of c and d. If $r \neq 0$, apply the division lemma to d and r.

Step 3: Write $d = er + r_1$ where $0 < r_1 < r$

Step 4: Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

- *Odd integers of the form **6q + 1, 6q + 3 or 6q + 5** shows that 6 is the divisor of given integer*
- *Any positive integer can be of the form 3m, 3m + 1, 3m + 2. Such that its cube would be of the form 9q + r.*

Fundamental Theorem of Arithmetic

Every composite number can be expressed as a product of primes and this expression is **unique**, except from the order in which the prime factors occur.

- *HCF is the lowest power of common prime and LCM is the highest power of primes.*
- *HCF (a, b) × LCM (a, b) = a × b.*
- *Any number ending with zero must have a factor of 2 and 5.*

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. L.C.M. of $2^3 \times 3^2$ and $2^3 \times 3^3$ is:

   (a) 2^3 (b) 3^3

   (c) $2^3 \times 3^3$ (d) $2^2 \times 3^2$

   [TERM 1, 2012]

2. If p and q are two co-prime numbers, then HCF (p, q) is:

   (a) p (b) q

   (c) pq (d) 1

   [TERM 1, 2013]

3. If $a = (2^2 \times 3^3 \times 5^4)$ and $b = (2^3 \times 3^2 \times 5)$, then HCF (a, b) is equal to:

   (a) 900 (b) 180

   (c) 360 (d) 540

   [TERM 1, 2013]

4. The HCF of two numbers is 27 and their LCM is 162, if one of the number is 54, find the other number.

   [TERM 1, 2017]

5. What is the HCF of the smallest prime number and the smallest composite number?

   [TERM 1, 2017]

2 Marks Questions

6. Show that 8^n cannot end with the digit zero for any natural number n.

   [TERM 1, 2011]

[TOPIC 2] Irrational Numbers, Terminating and Non-Terminating Recurring Decimals

Summary

Irrational Numbers

All real numbers which are not rational are called irrational numbers. $\sqrt{2}$, $\sqrt[3]{3}$, $-\sqrt{5}$ are some examples of irrational numbers.

There are decimals which are non-terminating and non-recurring decimal.

Example: 0.303003000300003…

Hence, we can conclude that

An irrational number is a non-terminating and non-recurring decimal and cannot be put in the form $\frac{p}{q}$ where p and q are both co-prime integers and $q \neq 0$.

Decimal Representation of Rational Numbers

Theorem: Let $x = \frac{p}{q}$ be a rational number such that $q \neq 0$ and prime factorization of q is of the form $2^n \times 5^m$ where m, n are non-negative integers then x has a decimal representation which terminates.

For example : $0.275 = \frac{275}{10^3} = \frac{5^2 \times 11}{2^3 \times 5^3} = \frac{11}{2^3 \times 5} = \frac{11}{40}$

Theorem: Let $x = \frac{p}{q}$ be a rational number such that $q \neq 0$ and prime factorization of q is not of the form $2^m \times 5^n$, where m, n are non-negative integers, then x has a decimal expansion which is non-terminating repeating.

For example : $\frac{5}{3} = 1.66666...$

Rational number	Form of prime factorisation of the denominator	Decimal expansion of rational number
$x = \frac{p}{q}$, where p and q are coprime and $q \neq 0$	q = $2^m 5^n$ where n and m are non-negative integers	terminating
	$q \neq 2^m 5^n$ where n and m are non-negative integers	non-terminating

- *If the denominator is of the form $2^m \times 5^n$ for some non negative integer m and n, then rational number has terminating decimal otherwise non terminating.*

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 2

1 Mark Questions

1. The prime factorization of the denominator of the rational number expressed as $46.\overline{123}$ is:

   (*a*) $2^m \times 5^n$ Where m and n are integers

   (*b*) $2^m \times 5^n$ Where m and n are positive integers

   (*c*) $2^m \times 5^n$ Where m and n are rational numbers

   (*d*) Not of the form $2^m \times 5^n$ where m and n are non-negative integers.

   [TERM 1, 2011]

2. The decimal expansion of $\frac{6}{1250}$ will terminate after how many places of decimal?

   (*a*) 1
   (*b*) 2
   (*c*) 3
   (*d*) 4

   [TERM 1, 2011]

[TOPIC 1] Zeroes of a Polynomial and Relationship between Zeroes and Coefficients of Quadratic Polynomials

Summary

Polynomials

An expression $p(x)$ of the form $p(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n$ where all $a_0, a_1, a_2, \ldots, a_n$, are real numbers and n is a non-negative integer, is called a polynomial.

The degree of a polynomial in one variable is the greatest exponent of that variable.

$a_0, a_1, a_2, \ldots, a_n$ are called the co-efficients of the polynomial $p(x)$.

a_n is called constant term.

Degree of a Polynomial

The exponent of the term with the highest power in a polynomial is known as its degree.

$f(x) = 8x^3 - 2x^2 + 8x - 21$ and $g(x) = 9x^2 - 3x + 12$ are polynomials of degree 3 and 2 respectively.

Thus, $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$ is a polynomial of degree n, if $a_0 \neq 0$.

On the basis of degree of a polynomial, we have following standard names for the polynomials.

A polynomial of degree 1 is called a **linear polynomial**. Example: $2x+3, \frac{1}{3}u+7$ etc.

A polynomial of degree 2 is called a **quadratic polynomial**. Example: $x^2 + 2x + 3$, $y^2 - 9$ etc.

A polynomial of degree 3 is called a **cubic polynomial**. Example: $x^3 + 7x - 3$, $-x^3 + x^2 + \sqrt{3}x$ etc.

A polynomial of degree 4 is called a **biquadratic polynomial**. Example: $3u^4 - 5u^3 + 2u^2 + 7$.

Value of a Polynomial

If $f(x)$ is a polynomial and α is any real number, then the real number obtained by replacing x by α in $f(x)$ is called the value of $f(x)$ at $x = \alpha$ and is denoted by $f(\alpha)$.

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. The graph of the polynomial $p(x)$ intersects the x-axis three times in distinct points, then which of the following could be an expression for $p(x)$:

   (a) $4 - 4x - x^2 + x^3$ (b) $3x^2 + 3x - 3$

   (c) $3x + 3$ (d) $x^2 - 9$

   [TERM 1, 2011]

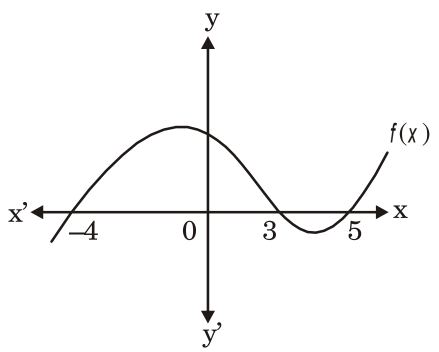
2. The polynomial whose zeroes are –5 and 4 is:

   (a) $x^2 - 5x + 4$ (b) $x^2 + 5x - 4$

   (c) $x^2 + x - 20$ (d) $x^2 - 9x - 20$

   [TERM 1, 2012]

3. In the given figure, the number of zeroes of the polynomial $f(x)$ are:



   (a) 1 (b) 2

   (c) 3 (d) 4

   [TERM 1, 2013]

[TOPIC 2] Problems on Polynomials

Summary

Zeros of a Polynomial

A real number α is a zero of polynomial $f(x)$ if $f(\alpha) = 0$.

The zero of a linear polynomial $ax + b$ is $-\frac{b}{a}$.

i.e. $-\frac{\text{Constant term}}{\text{Coefficient of } x}$

Geometrically zero of a polynomial is the point where the graph of the function cuts or touches x-axis.

When the graph of the polynomial does not meet the x-axis at all, the polynomial has no real zero.

Signs of Coefficients of a Quadratic Polynomial

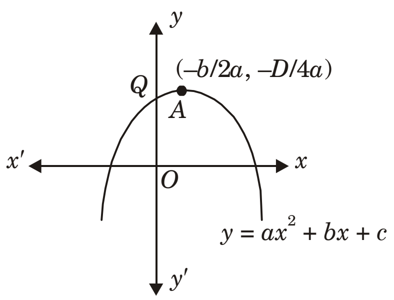
The graphs of $y = ax^2 + bx + c$ are given in figure. Identify the signs of a, b and c in each of the following:

(i) We observe that $y = ax^2 + bx + c$ represents a parabola opening downwards. Therefore, $a < 0$. We observe that the turning point $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ of the parabola is in first quadrant where $D = b^2 - 4ac$

$\therefore \quad -\frac{b}{2a} > 0$

$\Rightarrow \; -b < 0$

$\Rightarrow \; b > 0$ $\qquad [\because a < 0]$



Parabola $y = ax^2 + bx + c$ cuts y-axis at Q. On y-axis, we have $x = 0$.

Putting $x = 0$ in $y = ax^2 + bx + c$, we get $y = c$.

So, the coordinates of Q are (0, c). As Q lies on the positive direction of y-axis. Therefore, c > 0.

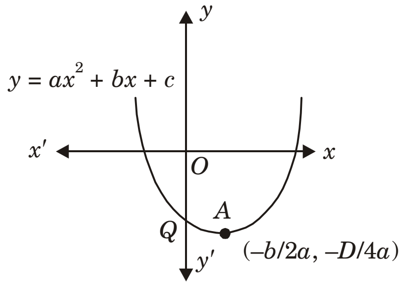
Hence, $a < 0$, $b > 0$ and $c > 0$.

(ii) We find that $y = ax^2 + bx + c$ represents a parabola opening upwards. Therefore, $a > 0$. The turning point of the parabola is in fourth quadrant.

$\therefore \quad -\frac{b}{2a} > 0$

$\Rightarrow \; -b > 0$

$\Rightarrow \; b < 0$



Parabola $y = ax^2 + bx + c$ cuts y-axis at Q and y-axis. We have $x = 0$. Therefore, on putting $x = 0$ in $y = ax^2 + bx + c$, we get $y = c$.

So, the coordinates of Q are (0, c). As Q lies on negative y-axis. Therefore, $c < 0$.

Hence, $a > 0$, $b < 0$ and $c < 0$.

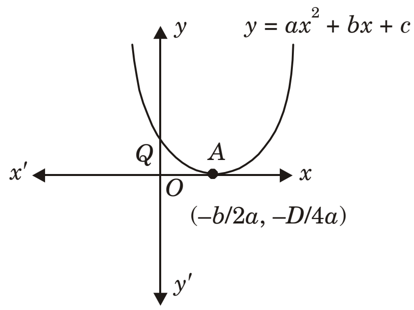
(iii) Clearly, $y = ax^2 + bx + c$ represents a parabola opening upwards.

Therefore, $a > 0$. The turning point of the parabola lies on positive direction of x-axis.

$\therefore \quad -\frac{b}{2a} > 0$

$\Rightarrow \; -b > 0$

$\Rightarrow \; b < 0$



The parabola $y = ax^2 + bx + c$ cuts y-axis at Q which lies on positive y-axis. Putting $x = 0$ in $y = ax^2 + bx + c$ we get $y = c$. So, the coordinates of Q are (0, c). Clearly, Q lies on OY.

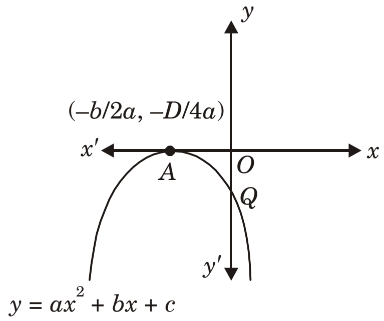
$\therefore \quad c > 0$.

Hence, $a > 0$, $b < 0$, and $c > 0$.

(iv) The parabola $y = ax^2 + bx + c$ opens downwards. Therefore, $a < 0$. The turning point $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ of the parabola is on negative x-axis,

$\therefore \quad -\frac{b}{2a} < 0$

$\Rightarrow \quad b < 0 \qquad [\because a < 0]$



Parabola $y = ax^2 + bx + c$ cuts y-axis at Q (0, c) which lies on negative y-axis. Therefore, $c < 0$.

Hence, $a < 0$, $b < 0$ and $c < 0$.

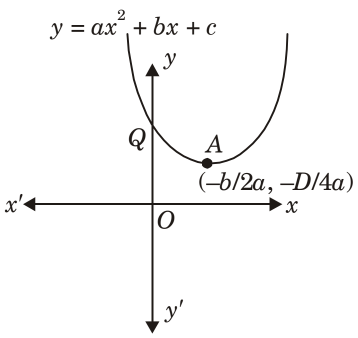
(v) We notice that the parabola $y = ax^2 + bx + c$ opens upwards. Therefore, $a > 0$.

Turning point $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ of the parabola lies in the first quadrant.

$\therefore \quad -\frac{b}{2a} > 0$

$\Rightarrow \quad \frac{b}{2a} < 0$

$\Rightarrow \quad b < 0 \qquad [\because a > 0]$



As Q (0, c) lies on positive y-axis. Therefore, $c > 0$.

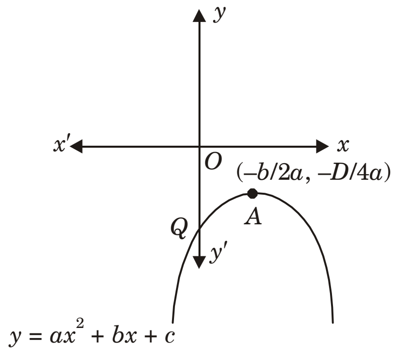
Hence, $a > 0$, $b < 0$ and $c > 0$.

(vi) Clearly, $a < 0$ Turning point $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ of the parabola lies in the fourth quadrant.

$\therefore \quad -\frac{b}{2a} > 0$

$\Rightarrow \quad \frac{b}{2a} < 0$

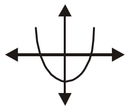
$\Rightarrow \quad b > 0 \qquad [\because a < 0]$

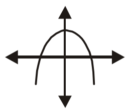


As Q (0, c) lies on negative y-axis. Therefore, $c < 0$.

Hence, $a < 0$, $b > 0$ and $c < 0$.

- ***The graph of quadratic polynomial is a parabola.***
- ***If a is +ve, graph opens upward.***
- ***If a is –ve, graph opens downward.***
- ***If D > 0, parabola cuts x-axis at two points i.e. it has two zeros.***





If D = 0, parabola touches x-axis at one point i.e. it has one zero.

If D < 0, parabola does not even touch x-axis at all i.e. it has no real zero.

Division Algorithm for Polynomials

Let $p(x)$ and $g(x)$ be polynomials of degree n and m respectively such that $m \leq n$. Then there exist unique polynomials $q(x)$ and $r(x)$ where $r(x)$ is either zero polynomial or degree of $r(x)$ < degree of $g(x)$ such that $p(x) = q(x) \, . \, g(x) + r(x)$.

$p(x)$ is dividend, $g(x)$ is divisor.

$q(x)$ is quotient, $r(x)$ is remainder

[TOPIC 1] Linear Equations (Two Variables)

Summary

Pair of Linear Equations in Two Variables

1. The equation of the form $ax = b$ or $ax + b = 0$, where a and b are two real numbers such that $x \neq 0$ and x is a variable is called a linear equation in one variable.
2. The general form of a linear equation in two variables is $ax + by + c = 0$ or $ax + by = c$ where a, b, c are real numbers and $a \neq 0$, $b \neq 0$ and x, y are variables.
3. The graph of a linear equation in two variables is a straight line.
4. The graph of a linear equation in one variable is a straight line parallel to x-axis for $ay = b$ and parallel to y-axis for $ax = b$, where $a \neq 0$.
5. A pair of linear equations in two variables is said to form a system of simultaneous linear equations.
6. The value of the variable x and y satisfying each one of the equations in a given system of linear equations in x and y simultaneously is called a solution of the system.

Graphical Method of Solution of a Pair of Linear Equations

1. Read the problem carefully to find the unknowns (variables) which are to be calculated.
2. Depict the unknowns by x and y etc.
3. Use the given conditions in the problem to make equations in unknown x and y.
4. Make the proper tables for both the equations.
5. Draw the graph of both the equations on the same set of axis.
6. Locate the co-ordinates of point of intersection of the graph, if any.
7. Coordinates of point of intersection will give us the required solution.

System of Two Simultaneous Linear Equations in X and Y

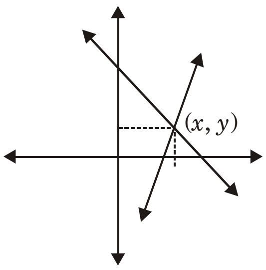
Consistent system: A system of two linear equations is said to be consistent if it has at least one solution.

Inconsistent system: A system of two linear equations is said to be inconsistent if it has no solution.

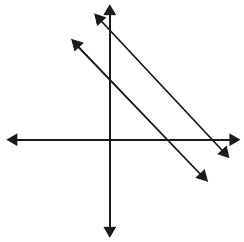
Let $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is a system of two linear equations.

The following cases occur :

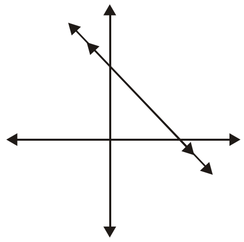
(i) if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, it has a unique solution. The graph of lines intersects at one point. The system is independent consistent.



(ii) if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. It has no solution. The graph of both lines is parallel to each other. The system is inconsistent



(iii) if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$. It has infinite many solutions. Every solution of one equation is a solution of other also. The graph of both equations is coincident lines. The system is dependent consistent.



[TOPIC 2] Different Methods to Solve Quadratic Equations

Summary

Methods of Solving Linear Equation

ELIMINATION METHOD (Eliminating One variable by making the coefficient equal to get the value of one variable and than put it in any equation to find other variable).

1. First multiply both the equations by some suitable non-zero constants to make the coefficients of one variable (either x or y) numerically equal.
2. Then add or subtract one equation from the other so that one variable gets eliminated. If you get an equation in one variable, go to step 3.
3. Solve the equation in one variable (x or y) so obtained to get its value.
4. Substitute this value of x (or y) in either of the original equations to get the value of the other variable.

- *If equations are of the following form:*

$$ax + by = cxy$$

$$dx + ey = fxy$$

Then, trivial solutions $x = 0$, $y = 0$ is one solution and the other solution can be obtained by elimination method.

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$$

SUBSTITUTION METHOD (Find the value of any one variable in terms of other and than use it to find other variable from the second equation).

1. Find the value of one variable, say y in terms of the other variable, i.e., x from either equation, whichever is convenient.
2. Substitute this value of y in the other equation, and reduce it to an equation in one variable, i.e., in terms of x, which can be solved.
3. Substitute the value of x (or y) obtained in Step 2 in the equation used in Step 1 to obtain the value of the other variable.

COMPARISON MEHTOD (Find the value of one variable from both the equation and equate them to get the value of other variable).

Let any pair of linear equations in two variables is of the form

$$a_1 x + b_1 y + c_1 = 0 \quad \text{...(i)}$$

$$a_2 x + b_2 y + c_2 = 0 \quad \text{...(ii)}$$

1. Find the value of one variable, say y in terms of other variable, i.e. x from equation (i), to get equation (iii).
2. Find the value of the same variable (as in step 1) in terms of other variable from equation (ii) to get equation (iv).
3. By equating the variable from equation (iii) and (iv) obtained in above two steps. We get the value of second variable.
4. Substituting the value of above said variable in equation (iii), we get the value of another variable.

CROSS MULTIPLICATION METHOD

Let the equation

$$a_1 x + b_1 y + c_1 = 0 \quad \text{...(i)}$$

$$a_2 x + b_2 y + c_2 = 0 \quad \text{...(ii)}$$

To obtain the values of x and y, we follow these steps:

1. Multiply Equation (i) by b_2 and (ii) by b_1, to get

$$b_2 a_1 x + b_2 b_1 y + b_2 c_1 = 0 \quad \text{...(iii)}$$

$$b_1 a_2 x + b_1 b_2 y + b_1 c_2 = 0 \quad \text{...(iv)}$$

2. Subtracting Equation (iv) from (iii), we get:

$$(b_2 a_1 - b_1 a_2)x + (b_2 b_1 - b_1 b_2)y + (b_2 c_1 - b_1 c_2) = 0$$

i.e. $(b_2 a_1 - b_1 a_2)x = b_1 c_2 - b_2 c_1$

So, $x = \dfrac{b_1 c_2 - b_2 c_1}{a_1 b_2 - a_2 b_1}$, if $a_1 b_2 - a_2 b_1 \neq 0$ ...(v)

3. Substituting this value of x in (i) or (ii), we get

$$y = \frac{c_1 a_2 - c_2 a_1}{a_1 b_2 - a_2 b_1} \quad \text{...(vi)}$$

We can write the solution given by equations (v) and (vi) in the following form:

$$\frac{x}{b_1 c_2 - b_2 c_1} = \frac{y}{c_1 a_2 - c_2 a_1} = \frac{1}{a_1 b_2 - a_2 b_1} \quad \text{...(vii)}$$

In remembering the above result, the following diagram may be helpful :

$$\overbrace{\begin{matrix} b_1 & & c_1 \\ & \times & \\ b_2 & & c_2 \end{matrix}}^{x} \quad \overbrace{\begin{matrix} c_1 & & a_1 \\ & \times & \\ c_2 & & a_2 \end{matrix}}^{y} \quad \overbrace{\begin{matrix} a_1 & & b_1 \\ & \times & \\ a_2 & & b_2 \end{matrix}}^{1} \quad \text{...(viii)}$$

For solving a pair of linear equations by this method, we will follow the following steps:

1. Write the given equations in the form (i) and (ii).
2. Taking the help of the diagram above, write equations as given in (viii).
3. Find x and y.

- ***For the equations like*** $a_1x + b_1y = c_1$ ***and*** $a_2x + b_2y = c_2$. ***The solution set can be calculated by using***

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{-1}{a_1b_2 - a_2b_1}$$

(Please note position of c_1 ***and*** c_2 ***with equality sign and subsequent change in the third term i.e. –1)***

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 2

2 Marks Question

1. Seema can row downstream 20km in 2 hours and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

[TERM 1, 2017]

3 Marks Questions

2. Solve for x and y.

$$99x + 101y = 1499$$

$$101x + 99y = 1501$$

[TERM 1, 2011]

3. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other they meet in 1 hour. What are the speeds of the two cars?

[TERM 1, 2011]

4. The sum of digits of a two-digit number is 7. If the digits are reversed, the new number decreased by 2 equals twice the original number. Find the number.

[TERM 1, 2012]

5. Solve for x and y

$$\frac{5}{x-1} + \frac{1}{y-2} = 2; \frac{6}{x-1} - \frac{3}{y-2} = 1; \begin{bmatrix} x \neq 1 \\ y \neq 2 \end{bmatrix}$$

[TERM 1, 2013]

6. Solve the following pair of equations:

$$49x + 51y = 499 \text{ and } 51x + 49y = 501$$

[TERM 1, 2014]

7. Solve the equation, $\frac{4}{x} - 3 = \frac{5}{2x+3}; x \neq 0, \frac{-3}{2}$, for x.

[TERM 1, 2014]

8. Solve for x and y:

$$\frac{2}{x-1} - \frac{1}{y-1} = 4$$

$$\frac{4}{x-1} - \frac{1}{y-1} = 10$$

[TERM 1, 2015]

9. Solve for x:

$$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0,\ x \neq 3, -\frac{3}{2}$$

[TERM 1, 2016]

10. Solve by elimination

$$3x = y + 5 \text{ and } 5x - y = 11$$

[TERM 1, 2016]

4 Marks Questions

11. The numerator of a fraction is 3 less than its denominator. If 1 is added to the denominator, the fraction is decreased by $\frac{1}{15}$. Find the fraction.

[TERM 1, 2012]

12. In a flight of 2800 km, an aircraft was slowed down due to bad weather. Its average speed is reduced by 100 km/h and time increased by 30 minutes. Find the original duration of the flight.

[TERM 1, 2012]

[TOPIC 1] Basic Concept of Quadratic Equations

Summary

Quadratic Equations

A polynomial of degree 2 (i.e. $ax^2 + bx + c$) is called a quadratic polynomial where $a \neq 0$ and a, b, c are real numbers.

Any equation of the form, $p(x) = 0$, where $p(x)$ is a polynomial of degree 2, is a quadratic equation. Therefore, $ax^2 + bx + c = 0$, $a \neq 0$ is called the standard form of a quadratic equation.

e.g. $2x^2 - 3x + 7$, $8x^2 + x - \sqrt{19}$

CLASSIFICATION OF A QUADRATIC EQUATION

It is classified into two categories:

(i) Pure quadratic equation (of the form $ax^2 + c = 0$ i.e., $b = 0$ in $ax^2 + bx + c = 0$)

e.g. $x^2 - 4 = 0$ and $3x^2 + 1 = 0$ are pure quadratic equations.

(ii) Affected quadratic equation (of the form $ax^2 + bx + c \neq 0$.

e.g. $x^2 - 2x - 8 = 0$ and $5x^2 + 3x - 2 = 0$ are affected quadratic equations.

ZEROS OF QUADRATIC POLYNOMIAL

For a quadratic polynomial $p(x) = ax^2 + bx + c$, those values of x for which $ax^2 + bx + c = 0$ is satisfied, are called zeros of quadratic polynomial $p(x)$, i.e. if $p(\alpha) = a\alpha^2 + b\alpha + c = 0$, then α is called the zero of quadratic polynomial.

ROOTS OF QUADRATIC EQUATION

If α, β are zeros of polynomial $ax^2 + bx + c$, then α, β are called roots (or solutions) of corresponding equation $ax^2 + bx + c = 0$ which implies that $p(\alpha) = p(\beta) = 0$.

i.e., $a\alpha^2 + b\alpha + c = 0$ and $a\beta^2 + b\beta + c = 0$.

Solution of a Quadratic Equation by Factorisation

Consider the following products.

$6 \times 0 = 0$; $-b \times 0 = 0$, $0 \times a = 0$

Above example illustrate that whenever the product is 0, at least one of the factors is 0.

- ***If a and b are numbers, then ab = 0, iff a = 0 or b = 0.***

Above principle is used in solving quadratic equation by factorisation. Let the given quadratic equation be $ax^2 + bx + c = 0$. Let the quadratic polynomial be expressed as product of two linear factors i.e. $(px + q)$ and $(rx + s)$, where p, q, r, s are real numbers and $p \neq 0$, $r \neq 0$,

Then, $ax^2 + bx + c = 0$

$\Rightarrow \quad (px + q)(rx + s) = 0$

$\therefore$ Either $(px + q) = 0$ or $(rx + s) = 0$

$px = -q$ or $rx = -s$

$x = -\dfrac{q}{p}$ or $x = -\dfrac{s}{r}$

Following steps are involved in solving a quadratic equation by factorisation.

- Transform the equation into standard form, if necessary.
- Factorise $ax^2 + bx + c$.
- Put each factor containing variable = 0.
- Solve each of the resulting equation

Solution of a Quadratic Equation by Completing the Square

Following steps are involved in solving a quadratic equation by quadratic formula

- Consider the equation $ax^2 + bx + c = 0$, where $a \neq 0$

- Dividing throughout by 'a', we get

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

- Add and subtract $\left(\frac{1}{2}\text{ coefficient of } x\right)^2$, we get

$$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2+\frac{c}{a}=0,$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}=\frac{b^2-4ac}{4a^2}$$

- If $b^2-4ac\geq 0$ taking square root of both sides, we obtain

$$x+\frac{b}{2a}=\frac{\pm\sqrt{b^2-4ac}}{2a}$$

Therefore $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

The Quadratic Formula: Quadratic equation, $ax^2+bx+c=0$, where a, b, c are real number and $a\neq 0$, has the roots as

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. The roots of the equation $x^2+x-p(p+1)=0$ where p is a constant, are

   (*a*) $p, p+1$ (*b*) $-p, p+1$

   (*c*) $p, -(p+1)$ (*d*) $-p, -(p+1)$

   [TERM 2, 2011]

2. The roots of the quadratic equation $2x^2-x-6=0$ are

   (*a*) $-2, \frac{3}{2}$ (*b*) $2, -\frac{3}{2}$

   (*c*) $-2, -\frac{3}{2}$ (*d*) $2, \frac{3}{2}$

   [TERM 2, 2012]

3. Solve the quadratic equation $2x^2+ax-a^2=0$ for x.

   [TERM 2, 2014]

4. If $x=-\frac{1}{2}$, is a solution of the quadratic equation $3x^2+2kx-3=0$, find the value of k.

   [TERM 2, 2015]

2 Marks Questions

5. Form a quadratic polynomial whose zeroes are $\frac{3-\sqrt{3}}{5}$ and $\frac{3+\sqrt{3}}{5}$

   [TERM 2, 2013]

6. Solve the following quadratic equation for :

   $4\sqrt{3}x^2+5x-2\sqrt{3}=0$

   [TERM 2, 2013]

7. Find the quadratic polynomial whose zeroes are $\sqrt{3}+\sqrt{5}$ and $\sqrt{5}-\sqrt{3}$.

   [TERM 2, 2014]

8. Solve the following quadratic equation for x:

   $4x^2-4a^2x+\left(a^4-b^4\right)=0$

   [TERM 2, 2015]

9. Solve the following quadratic equations for x:

   $4x^2+4bx-\left(a^2-b^2\right)=0$

   [TERM 2, 2015]

10. If $x=\frac{2}{3}$ and $x=-3$ are roots of the quadratic equation $ax^2+7x+b=0$, find the values of a and b.

    [TERM 2, 2016]

11. Find the roots of the quadratic equation $\sqrt{2}x^2+7x+5\sqrt{2}=0$

    [TERM 2, 2017]

[TOPIC 2] Roots of a Quadratic Equation

Summary

Nature of Roots

In previous section, we have studied that the roots of the equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

A quadratic equation $ax^2 + bx + c = 0$ has

- **Two distinct real roots if $b^2 - 4ac > 0$.**
  If $b^2 - 4ac > 0$, we get two distinct real roots $-\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}$ and $-\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$
- **Two equal roots, if $b^2 - 4ac = 0$.**
  If $b^2 - 4ac > 0$, then $x = -\frac{b \pm 0}{2a}$
  i.e. $x = -\frac{b}{2a}$
  So, the roots are both $-\frac{b}{2a}$
- **No real roots, if $b^2 - 4ac < 0$**
  If $b^2 - 4ac < 0$, then there is no real number whose square is $b^2 - 4ac$.
- ***($b^2 - 4ac$) determines whether the quadratic equation $ax^2 + bx + c = 0$ has real roots or not, hence ($b^2 - 4ac$) is called the discriminant of quadratic equation. It is denoted by D.***

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 2

1 Mark Question

1. If the quadratic equation $px^2 - 2\sqrt{5}px + 15 = 0$ has two equal roots,
   Then find the value of p.
   [TERM 2, 2015]

2 Marks Questions

2. Find the value of p so that the quadratic equation $px(x - 3) + 9 = 0$ has two equal roots.
   [TERM 2, 2017]
3. Find the value of p for which the roots of the equation $px(x - 2) + 6 = 0$, are equal.
   [TERM 2, 2012]
4. Find the values of p for which the quadratic equation $4x^2 + px + 3 = 0$ has equal roots.
   [TERM 2, 2014]
5. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.
   [TERM 2, 2016]
6. Find the value of k for which the equation $x^2 + k(2x + k - 1) + 2 = 0$ has real and equal roots.
   [TERM 2, 2017]

3 Marks Questions

7. Find that non-zero value of k, for which the quadratic equation $kx^2 + 1 - 2(k - 1)x + x^2 = 0$ has equal roots. Hence find the roots of the equation.
   [TERM 2, 2015]
8. If $ad \neq bc$, then prove that the equation $(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$ has no real roots.
   [TERM 2, 2017]
9. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots then show that $c^2 = a^2(1 + m^2)$.
   [TERM 2, 2017]
10. Is it possible to design a rectangular park of perimeter 80m and area 400m^2? If so find its length and breadth.
    [TERM 2, 2017]

4 Marks Questions

11. Find the values of k for which the quadratic equation $(k + 4)x^2 + (k + 1)x + 1 = 0$ has equal roots. Also find these roots.
    [TERM 2, 2014]

[TOPIC 1] Arithmetic Progression

Summary

Sequence and Series

Sequence: A sequence is an arrangement of number in a definite order, according to a definite rule.

Terms: Various numbers occurring in a sequence are called terms or element.

Consider the following lists of number:

3, 6, 9, 12, ...

4, 8, 12, 16, ...

–3, –2, –1, 0, ...

In all the list above, we observe that each successive terms are obtained by adding a fixed number to the preceding terms. Such list of numbers is said to form on **Arithmetic Progression (AP).**

Arithmetic Progression: An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.

This fixed number is called the **common difference (d)** of the A.P.

Common difference can be positive, negative or zero.

Let us denote first term of A.P. by a or t, second term by a_2 or t_2 and nth term by a_n or t_n & the common difference by d. Then the A.P. becomes

$a_1,\quad a_2,\quad a_3 \quad \ldots\ldots\ldots\ldots \quad a_n$

where $a_2 - a_1 = d$

or $a_2 = a_1 + d$

similarly $a_3 = a_2 + d$

- $\therefore$ ***In general,*** $a_n - a_{n-1} = d$

  or $a_n = a_{n-1} + d$

Thus $a,\ a + d,\ a + 2d,\ \ldots\ldots\ldots$

forms an A.P. whose first term is 'a' & common difference is 'd'

This is called general form of an A.P.

Finite A.P. : An A.P. containing finite number of terms is called finite A.P.

e.g. 147, 149, 151 .... 163.

Infinite A.P. : An A.P. containing infinite terms is called infinite A.P.

e.g. 6, 9, 12, 15 ....

nth Term of an A.P.

Let a_1, a_2, a_3 ..... be an A.P., with first term as a, and common difference as d.

$$\begin{aligned}
\text{First term is } &= a && \text{(i)}\\
\text{Second term } (a_2) &= a + d && \text{(ii)}\\
&= a + (2-1)d\\
\text{Third term } (a_3) &= a_2 + d && \text{(iii)}\\
&= a + d + d && \text{[from (i)]}\\
&= a + 2d\\
\text{or} \quad &= a + (3-1)d\\
\text{Fourth term} \quad a_4 &= a_3 + d\\
\text{or} \quad &= a + 2d + d && \text{[from (iii)]}\\
&= a + 3d\\
&= a + (4-1)d\\
\therefore \quad \text{nth term } a_n &= a + (n-1)d
\end{aligned}$$

- ***The nth term of the A.P. with first term a & common difference d is given by*** $a_n = a + (n - 1)d$

a_n is also called as general term of an A.P.

If there are P terms in the A.P. then a_p represents the last term which can also be denoted by l.

TO FIND nth TERM FROM THE END OF AN A.P.

Consider the following A.P. $a, a + d, a + 2d, \ldots (l - 2d), (l - d), l$

where ℓ is the last term

$$\begin{aligned}
\text{last term} \quad l &= l - (1-1)d\\
2^{nd} \text{ last term} \quad l - d &= l - (2-1)\,d\\
3^{rd} \text{ last term} \quad l - 2d &= l - (3-1)d\\
&\ldots\ldots\ldots\ldots\\
&\ldots\ldots\ldots\ldots
\end{aligned}$$

- ***nth term from the end =*** $l - (n - 1)\,d$

CONDITION FOR TERMS TO BE IN A.P.

If three numbers $a, b, c,$ in order are in A.P. Then,

$b - a$ = common difference = $c - b$

$\Rightarrow\ b - a = c - b$

$\Rightarrow\ 2b = a + c$

- ***a, b, c are in A.P. iff***

[TOPIC 2] Sum of n Terms of an A.P.

Summary

Sum of n Terms of an A.P.

Let a be the first term and d be the common difference of an A.P. l is the last term where $l = a + (n - 1)\, d$.

Sum of first n terms of the given A.P. is given by

$$S_n = a + (a + d) + (a + 2d) + ...(l - 2d) + (l - d) + l \quad .(i)$$

Writing in reverse order

$$S_n = l + (l - d) + (l - 2d) + ...(a + 2d) + (a + d) + a \quad .(ii)$$

Adding (i) and (ii) we get

$$2S_n = \underbrace{(a+l) + (a+l) + (a+l) + ... + (a+l)}_{n \text{ times}}$$

$$2S_n = n(a + l)$$

$$S_n = \frac{n}{2}(a + l) = \frac{n}{2}[a + a + (n-1)\,d]$$

$[\because\ l = a + (n - 1)d]$

$$S_n = \frac{n}{2}[2a + (n-1)\,d]$$

$$S_n = \frac{n}{2}(a + a_n)$$

where $a_n = a + (n - 1)d$

Selection of Terms in A.P.

Some times certain number of terms in A.P. are required. The following ways of selecting terms are convenient.

Number of terms	Terms	common difference
3	$a - d, a, a + d$	d
4	$a - 3d, a - d, a + d, a + 3d$	$2d$
5	$a - 2d, a - d, a, a + d, a + 2d$	d
6	$a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$	$2d$

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 2

1 Mark Question

1. If the n^{th} term of an A.P. is $(2n + 1)$, then the sum of its first three terms is

   (*a*) $6n + 3$ (*b*) 15

   (*c*) 12 (*d*) 21

   [TERM 2, 2012]

2 Marks Questions

2. The first and the last terms of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference.

   [TERM 2, 2014]

3. In an AP, if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the AP, where S_n denotes the sum of first terms.

   [TERM 2, 2015]

4. How many terms of the A.P. 18, 16, 14 ... be taken so that their sum is zero?

   [TERM 2, 2016]

5. How many terms of A.P 27, 24, 21, ... should be taken so that their sum is zero (0)?

   [TERM 2, 2017]

6. Find the sum of first 8 multiple of 3.

   [DELHI 2018]

3 Marks Questions

7. Find the sum of all multiples of 7 lying between 500 and 900.

   [TERM 2, 2012]

8. Find the number of terms of the A.P. $18, 15\frac{1}{2}, 13, ..., -49\frac{1}{2}$ and find the sum of all its terms.

   [TERM 2, 2013]

[TOPIC 1] Distance between two Points and Section Formula

Summary

Coordinates of a Point

Location of the position of a point on a plane requires a pair of co-ordinate axes. The distance of a point from the x-axis is called its y-coordinate, or ordinate. The distance of a point from the y-axis is called its x-co-ordinate or abscissa. The co-ordinates of a point on the x-axis are of the form $(x, 0)$ and of a point on the y-axis are of the form $(0, y)$.

DISTANCE FORMULA

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by the formula

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

which is called the distance formula.

- ***In particular, the distance of a point P(x, y) from the origin O(0, 0) is given by***

$$OP = \sqrt{x^2 + y^2}$$

COLLINEAR POINTS

Three points A, B, C are said to be collinear if they lie on the same straight line.

Test For Collinearity of ThreePoints

In order to show that three given points A, B, C are collinear, we find distances AB, BC and AC. If the sum of any two of these distances is equal to the third distance, then the given points are collinear.

- *In a triangle , sum of any two sides is greater than the third side.*
- *Any point on x-axis is of the form (x, 0).*
- *Any point on y-axis is of the form (0, y).*
- *Circumcentre of a triangle is equidistant from its three vertices.*

Section Formula

Section formula : The coordinates of the point $P(x, y)$ which divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio m : n are given by

$$x = \frac{mx_2 + nx_1}{m+n}, \ y = \frac{my_2 + ny_1}{m+n}$$

Midpoint formula : The coordinates of the midpoint M of a line segment AB with end points $A(x_1, y_1)$ and $B(x_2, y_2)$ are $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

- *Diagonals of a parallelogram bisect each other.*

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. The point P which divides the line segment joining the points $A\,(2, -5)$ and $B\,(5, 2)$ in the ratio 2 : 3 lies in the quadrant

   (*a*) I (*b*) II

   (*c*) III (*d*) IV

   [TERM 1, 2011]

2. The mid-point of segment AB is the point P(0, 4). If the coordinates of B are $(-2,3)$ then the coordinates of A are

   (*a*) $(2,5)$ (*b*) $(-2,-5)$

   (c) $(2,9)$ (*d*) $(-2,11)$

   [TERM 1, 2011]

3. The distance of the point $(-3,4)$ from the x-axis is

   (*a*) 3 (*b*) –3

   (*c*) 4 (d) 5

   [TERM 1, 2012]

[TOPIC 2] Centroid and Area of Triangle

Summary

Centroid of a Triangle

The coordinates of the centroid of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by $\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}$.

Area of a Triangle

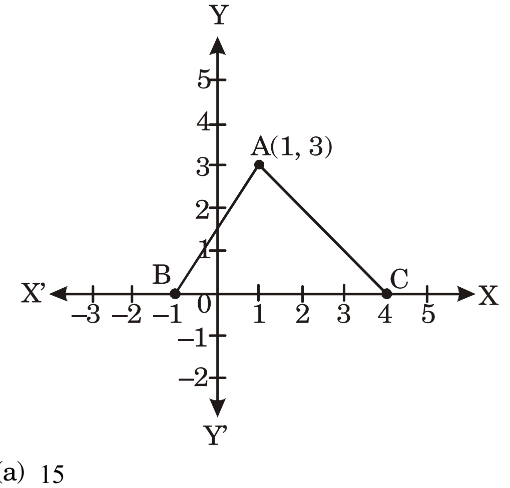
The area of a ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by area

$$(\Delta ABC) = \left|\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}\right|.$$

PREVIOUS YEARS' EXAMINATION QUESTIONS TOPIC 2

1 Mark Questions

1. In Fig, find the area of triangle ABC (in sq. units) is:



(a) 15

(b) 10

(c) 7.5

(d) 2.5

[TERM 1, 2013]

2. If the points $A(x, 2)$, $B(-3, -4)$ and $C(7, -5)$ are collinear, then the value of x is:

(a) –63 (b) 63

(c) 60 (d) –60

[TERM 1, 2014]

2 Marks Question

3. Find the relation between x and y if the points A(x, y), B(–5, 7) and $C(-4, 5)$ are collinear.

[TERM 1, 2015]

3 Marks Questions

4. If (3,3), (6,y), (x,7) and (5,6) are the vertices of a parallelogram taken in order, find the values of x and y

[TERM 1, 2011]

5. Find the value of k, if the points P(5,4), Q(7,k) and R(9,-2) are collinear.

[TERM 1, 2011]

6. Find the area of the quadrilateral ABCD whose vertices are A(–3, –1), B(–2, –4), C(4, –1) and D(3, 4).

[TERM 1, 2012]

7. If the points A(x, y), B(3, 6) and C(-3, 4) are collinear, show that x – 3y + 15 = 0.

[TERM 1, 2012]

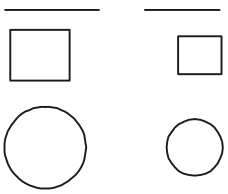
Summary

Similar Triangles

Similar figures: Geometric figures which have the same shape but different sizes are known as similar figures.

Illustrations:

1. Any two line-segments are similar
2. Any two squares are similar
3. Any two circles are similar



Two congruent figures are always similar but two similar figures need not be congruent.

Similar polygons: Two polygons of the same number of sides are said to be similar if

(i) their corresponding angles are equal (i.e., they are equiangular) and

(ii) their corresponding sides are in the same ratio (or proportion)

Similar triangles: Since triangles are also polygons, the same conditions of similarity are applicable to them.

Two triangles are said to be similar if

(i) their corresponding angles are equal and

(ii) their corresponding sides are in the same ratio (or proportion).

BASIC-PROPORTIONALITY THEOREM (Thales theorem)

Theorem 1 : If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Theorem 2 : (Converse of BPT theorem) If a line divides any two sides of a triangle in the same ratio, prove that it is parallel to the third side.

Criteria for Similarity of Two Triangles

Two triangles are said to be similar if (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio (or proportional).

Thus, two triangles ABC and $A'B'C'$ are similar if

(i) $\angle A = \angle A'$, $\angle B = \angle B'$, $\angle C = \angle C$ and

(ii) $\dfrac{AB}{A'B'} = \dfrac{BC}{B'C'} = \dfrac{CA}{C'A'}$

In this section, we shall make use of the theorems discussed in earlier sections to derive some criteria for similar triangles which in turn will imply that either of the above two conditions can be used to define the similarity of two triangles.

CHARACTERISTIC PROPERTY 1 (AAA SIMILARITY)

Theorem 3: If in two triangles, the corresponding angles are equal, then the triangles are similar.

CHARACTERISTIC PROPERTY 2 (SSS SIMILARITY)

Theorem 4: If the corresponding sides of two triangles are proportional, then they are similar.

CHARACTERISTIC PROPERTY 3 (SAS SIMILARITY)

Theorem 5: If one angle of a triangle is equal to one angle of the other and the sides including these angles are proportional than the two triangles are similar.

Areas of Similar Triangles

Theorem 6:The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Pythagoras Theorem

Theorem 7:In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Theorem 8: (Converse of Pythagoras theorem) In a triangle if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

- *The mid-point of the hypotenuse of a right triangle is equidistant from the vertices.*

Summary

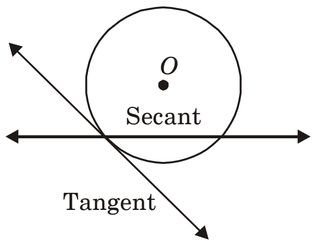
Circles

DEFINITIONS

Secant: A line, which intersects a circle in two distinct points, is called a secant.

Tangent: A line meeting a circle only in one point is called a tangent to the circle at that point.

The point at which the tangent line meets the circle is called the point of contact.



Length of tangent: The length of the line segment of the tangent between a given point and the given point of contact with the circle is called the length of the tangent from the point to the circle.

- ***There is no tangent passing through a point lying inside the circle.***
- ***There is one and only one tangent passing through a point lying on a circle.***
- ***There are exactly two tangents through a point lying outside a circle.***

Theorem 1 : The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Theorem 2 : The lengths of tangents drawn from an external point to a circle are equal.

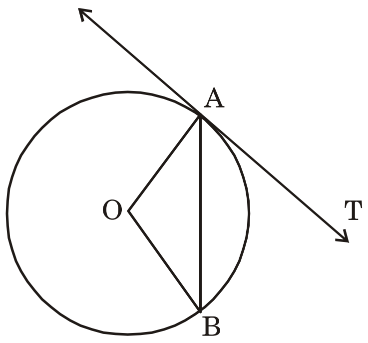
- ***The centre lies on the bisector of the angle between the two tangents.***

PREVIOUS YEARS'

EXAMINATION QUESTIONS

1 Mark Questions

1. In the given figure, O is the centre of a circle, AB is a chord and AT is the tangent at A. If $\angle AOB = 110°$, then $\angle BAT$ is equal to



(*a*) 100° (*b*) 40°
(*c*) 50° (*d*) 90°

[TERM 2, 2011]

2. The radii of two circles are 4 cm and 3 cm respectively. The diameter of the circle having area equal to the sum of the areas of the two circles (in cm) is

(*a*) 5
(*b*) 7
(c) 10
(*d*) 14

[TERM 2, 2011]

3. From a point Q, 13 cm away from the centre of a circle, the length of tangent PQ to the circle is 12 cm. The radius of the circle (in cm) is

(*a*) 25
(*b*) $\sqrt{313}$
(*c*) 5
(d) 1

[TERM 2, 2012]

[TOPIC 1] Construction of a Line Segment

Summary

Division of a Line Segment in a given Ratio

In this chapter we shall study some constructions by using the knowledge of the earlier constructions done in previous class.

Solved Examples (Three Marks Each)

Illustration 1

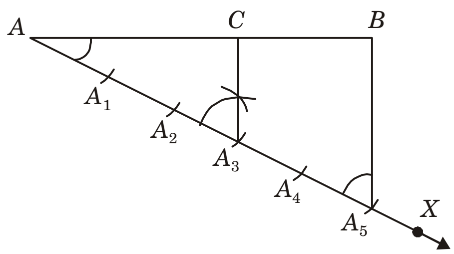
Question:

To divide a line segment in a given ratio 3 : 2.

Solution:

Given a line segment AB, we want to divide it in the ratio 3 : 2.

Steps of construction:



1. Draw any ray AX, making an acute angle with AB.
2. Locate 5 points A_1, A_2, A_3, A_4 and A_5 on AX so that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.
3. Join BA_5.
4. Through the point A_3, draw *a* line parallel to A_5B (by making an angle equal to $\angle AA_5B$) intersecting AB at the point C

   Then, $AC : CB = 3 : 2$

Justification:

Since A_3C is parallel to A_5B, therefore,

$$\frac{AA_3}{A_3A_5} = \frac{AC}{CB} \text{ (By Basic proportionality theorem)}$$

By construction,

$$\frac{AA_3}{A_3A_5} = \frac{3}{2}.$$

Therefore,

$$\frac{AC}{CB} = \frac{3}{2}.$$

This shows that C divides AB in the ratio 3 : 2

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 1

2 Marks Question

1. Draw a line segment of length 8cm and divide it internally in the ratio 4 : 5.

[TERM 2, 2017]

Solutions

1. The steps to divide a line segment of length 8cm in the ratio of 4 : 5 are as follows:

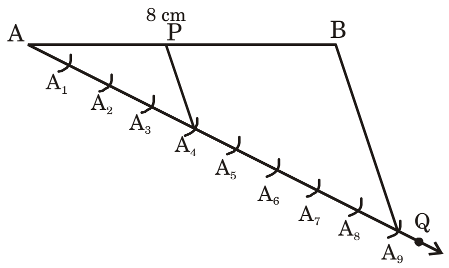
   Step 1: Draw a line segment AB of 8cm and draw a ray from A making an acute angle with line segment AB. [½]

   Step 2: Make 9 points, $A_1, A_2, A_3, A_4, \ldots, A_9$ on AQ such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 \ldots\ldots A_8A_9$

   Step 3: Join BA_9. [½]

   Step 4: Through the point , draw a line parallel to BA_9 by making an angle equal to $\angle AA_9B$ at A_4 intersecting AB at point P.

   P is the point dividing line segment AB in the ratio of 4 : 5.



[1]

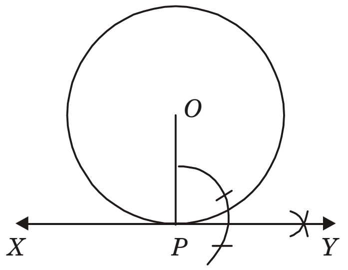
[TOPIC 2] Construction of a Tangent to a Circle from a Point Outside it.

Summary

Construction of Tangents From a Point Outside the Circle

WHEN CENTRE IS GIVEN

When point of tangency is on the circle.



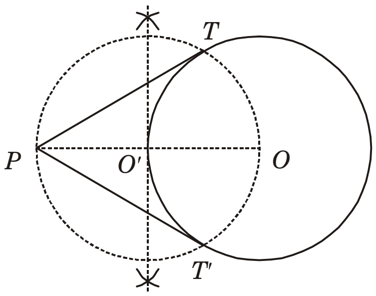
Given : A circle with centre O.

Required : To draw a tangent from point P on the circle.

Steps of construction:

1. Take a point O on the plane of the paper and draw a circle of given radius.
2. Take a point P on the circle.
3. Join OP.
4. Construct $\angle OPX = 90°$.
5. Produce XP to Y to get XPY as the required tangent.

When point of tangency is outside the circle



Given : A circle with centre O.

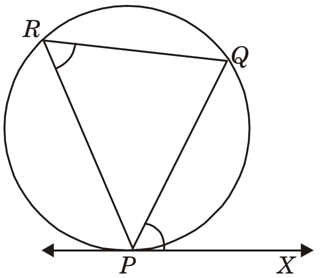
Required : To draw a tangent from an external point i.e., P.

Steps of construction:

1. Join the centre O of the circle to the given external point i.e., P.
2. Draw $\perp$ bisector of OP, intersecting OP at O'.
3. Taking O' as centre and $OO' = PO'$ as radius, draw a circle to intersect the given circle at T and T'.
4. Join PT and PT' to get the required tangents as PT and PT'.

WHEN CENTRE IS NOT GIVEN

When point of tangency is on the circle.



Given : A circle and a point P on it.

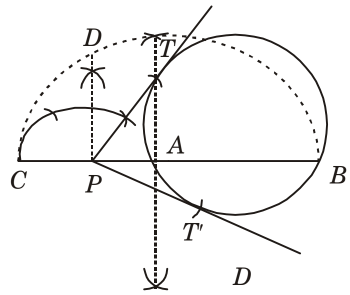
Required : To draw a tangent at P without using centre of the circle.

Steps of construction:

1. Draw any chord PQ of the circle through P as in figure.
2. Take any point R on the major arc PQ and join PR and QR.
3. Construct $\angle QPX$ equal to $\angle PRQ$.

   Then PX is the required tangent at P to the circle.

When point of tangency is outside the circle.



Given : A circle and a point P outside it.

Required : To draw a tangent from point P without using the centre.

Steps of construction:

1. Let P be the external point from where the tangents are to be drawn to the given circle.
2. Through P draw a secant PAB to intersect the circle at A and B.
3. Produce AP to a point C such that $AP = PC$.
4. Draw a semi-circle with BC as diameter.
5. Draw $PD \perp CB$, intersecting the semi-circle at D.
6. With P as centre and PD as radius draw arcs to intersect the given circle at T and T'.
7. Joint PT and PT'. PT and PT' are the required tangents.

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 2

3 Marks Question

1. Draw a right triangle ABC in which $AB = 6$ cm, $BC = 8$ cm and $\angle B = 90°$. Draw BD perpendicular from B on AC and draw a circle passing through the points B, C and D. Construct tangents from A to this circle.

[TERM 2, 2014]

4 Marks Questions

2. Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of to each other.

[TERM 2, 2016]

3. Draw two concentric circles of radii 3 cm and 5 cm. Construct a tangent to smaller circle from a point on the larger circle. Also measure its length.

[TERM 2, 2016]

Solutions

1. Follow the given steps to construct the figure.

Step 1:Draw a line AB = 6 cm segment from point B, draw a ray making an angle of 90° with AB. Now with B as center and radius 8 cm draw an arc cutting the ray at point C. Join AC, to form ΔABC. Thus, ΔABC is created. [½]

Step 2: Bisect BC and name the midpoint of BC as E. So, the center of circle is E. [½]

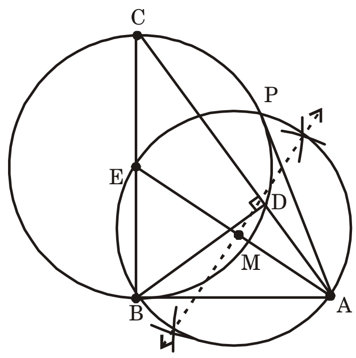
Step 3: Join points A and E. Bisect AE and name the midpoint of AE is M. [½]

Step 4: With M as centre and ME as radius, draw a circle. [½]

Step 5: Let it intersect given circle at B and P.

Step 6: Join AP and AB.

Here, AB and AP are the required tangents to the circle from A.



[1]

2. Steps of construction:

(i) Take a point O on the plane of the paper and draw a circle of radius

(ii) Produce OA to B such that OA = AB = 4 cm [½]

(iii) Draw a circle with center at A and radius AB.

(iv) Suppose it cuts the circle drawn in step (i) at P and Q. [½]

[TOPIC 3] Construction of a triangle Similar to a given Triangle

Summary

Some Constructions of Triangles

1. **Rules of Congruency of Two Triangles**
   (i) **SAS :** Two triangles are congruent, if any two sides and the included angle of one triangle are equal to any two sides and the included angle of the other triangle.
   (ii) **SSS :** Two triangles are congruent if the three sides of one triangle are equal to the three sides of the other triangle.
   (iii) **ASA :** Two triangles are congruent if any two angles and the included side of one triangle are equal to the two angles and the included side of the other triangle.
   (iv) **RHS :** Two right triangles are congruent if the hypotenuse and a side of one triangle are respectively equal to the hypotenuse and a side of the other triangle.
2. **Uniqueness of a Triangle**
   A triangle is unique if
   (i) two sides and the included angle is given
   (ii) three sides and angle is given
   (iii) two angles and the included side is given and,
   (iv) in a right triangle, hypotenuse and one side is given.

Note : At least three parts of a triangle have to be given for constructing it but not all combinations of three parts are sufficient for the purpose.

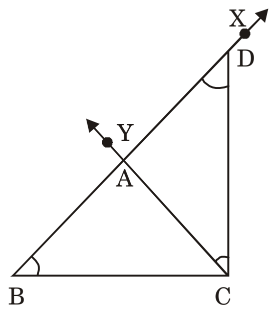
Basic Constructions of Triangles:

Statement 1 : To construct a triangle, given its base, a base angle and sum of other two sides.

Given : Base BC, a base angle, say $\angle B$ and the sum AB + AC of the other two sides of a triangle ΔABC

Required : To construct a ΔABC.

Steps of construction



1. Draw the base BC and at the point B make an angle, say XBC equal to the given angle.
2. Cut a line segment BD equal to AB + AC from the ray BX.
3. Join DC and make an angle DCY equal to $\angle BDC$.
4. Let CY intersect BX at A (see fig.)

Then, ABC is the required triangle.

Note : The construction of the triangle is not possible if the sum $AB + AC \leq BC$.

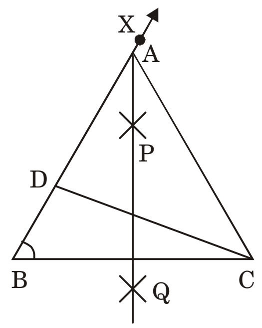
Statement 2 : To construct a triangle given its base, a base angle and the difference of the other two sides.

Given : The base BC, a base angle, say Z B and the difference of other two sides AB – AC or AC – AB.

Require : Construct the triangle ABC.

Case (i) : Let AB > AC that is AB – AC is given.

Steps of Construction :

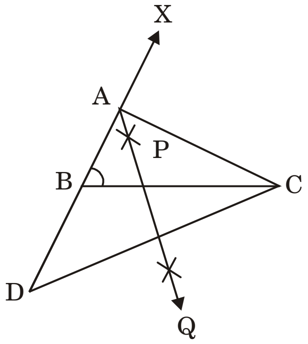


1. Draw the base BC and at point B make an angle say XBC equal to the given angle.
2. Cut the line segment BD equal to AB – AC from ray BX.

3. Join DC and draw the perpendicular bisector, say PQ of DC.
4. Let it intersect BX at a point A. Join AC (see fig.) I hen ABC is the required triangle.

Case (ii) : Let AB < AC that is AC – AB is given.

Steps of Construction :



1. Draw the base BC and at B make an angle XBC equal to the given angle.
2. Cut the line segment BD equal to AC – AB from the line BX extended on opposite side of line segment BC.
3. Join DC and draw the perpendicular bisector, say PQ of DC.
4. Let PQ interseel BX at A. Join AC (see fig.) Then, ABC is the required triangle.

Statement 3 : To construct a triangle, given its perimeter and its two base angles.

Given : The base angles, say ∠B and ∠C and BC + CA + AB.

Required : Construct the triangle ABC.

Steps of Construction :

1. Draw a line segment, say XY equal to BC + CA – FAB.
2. Make angle LXY equal to ∠B and MYX equal to ∠C.
3. Bisect ∠LXY and ∠MYX. Let these bisectors intersect at a point A. (see fig. (i))

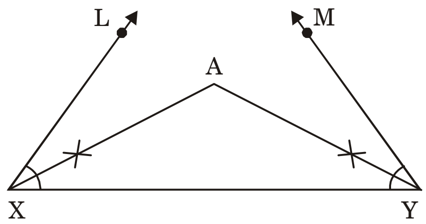


Fig (i)

4. Draw perpendicular bisectors PQ of AX and RS of AY.
5. Let PQ intersect XY at B and RS intersect XY at C. join AB and AC. (see fig. (ii))

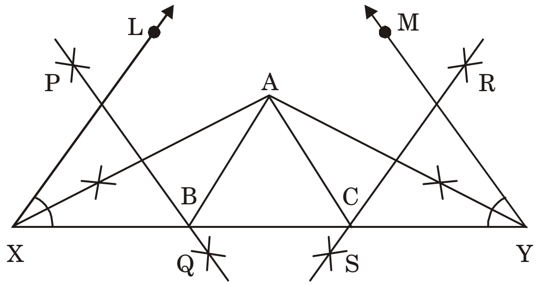


Fig (ii)

Then ABC is the required triangle.

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 3

3 Marks Questions

1. Draw a triangle ABC in which AB = 5 cm, BC = 6 cm and $\angle ABC = 60°$. Then construct a triangle whose sides are $\frac{5}{7}$ times the corresponding sides of ΔABC.

[TERM 2, 2011]

2. Draw a triangle ABC with BC = 7 cm, ∠B = 45° and ∠C = 60°. Then construct another triangle, whose sides are $\frac{3}{5}$ times the corresponding sides of ΔABC.

[TERM 2, 2012]

3. Construct a triangle with sides 5 cm, 4 cm and 6 cm. Then construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of first triangle.

[TERM 2, 2013]

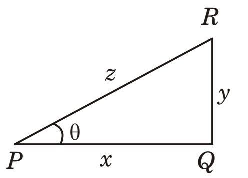
[TOPIC 1] Trigonometric Ratios

Summary

Trigonometry

TRIGONOMETRY: It is that branch of mathematics, which deals with the measurement of angles and the problems related with angles.

TRIGONOMETRIC RATIOS (T-RATIOS)



Let $\angle RPQ = \theta$ be the given angle of a right-angled ΔPQR.

In right-angled ΔPQR, let base = $PQ = x$ units, Perpendicular = $QR = y$ units and hypotenuse = $PR = z$ units.

Trigonometric ratios for θ are defined as below:

(i) sine $\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{y}{z}$, and is written as sin θ.

(ii) cosine $\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{x}{z}$, and is written as cos θ.

(iii) tangent $\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{y}{x}$, and is written as tan θ.

(iv) cosecant $\theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{z}{y}$, and is written as cosec θ.

(v) secant $\theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{z}{x}$, and is written as sec θ.

(vi) cotangent $\theta = \frac{\text{base}}{\text{perpendicular}} = \frac{x}{y}$, and is written as cot θ.

RECIPROCAL RELATION

We have

(i) $\text{cosec}\ \theta = \frac{1}{\sin\theta}$

(ii) $\sec\theta = \frac{1}{\cos\theta}$

(iii) $\cot\theta = \frac{1}{\tan\theta}$

- ***The value of each of the trigonometric ratios of an angle does not depend on the size of the triangle. It only depends on the angle.***

POWER OF T-RATIOS

We write $(\sin\theta)^2 = \sin^2\theta$; $(\sin\theta)^3 = \sin^3\theta$; $(\cos\theta)^3 = \cos^3\theta$; etc.

QUOTIENT RELATION OF T-RATIOS

Theorem 1: For any acute angle θ, prove that

(i) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

(ii) $\cot\theta = \frac{\cos\theta}{\sin\theta}$

SQUARE RELATION

Theorem 2: For any acute angle θ, prove that

(i) $\sin^2\theta + \cos^2\theta = 1$;

(ii) $1 + \tan^2\theta = \sec^2\theta$;

(iii) $1 + \cot^2\theta = \text{cosec}^2\theta$.

- ***The value of sin θ increases from 0 to 1 as the angle θ increases from 0º to 90°.***
- ***The value of cos θ decreases from 1 to 0 as the angle θ increases from 0º to 90°.***

VALUES OF ALL THE TRIGONOMETRIC RATIOS OF 0°, 30°, 45°, 60° AND 90°.

θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	n. d.
cosecθ	n. d.	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	n. d.
cot θ	n. d.	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

If A and B are two complementary acute angles, i.e., $A + B = 90º$, then we have

sin A = sin (90° – B) = cos B

cos A = cos (90° – B) = sin B

tan A = tan (90° – B) = cot B

cosec A = cosec (90° – B) = sec B

sec A = sec (90° – B) = cosec B

cot A =cot (90° – B) = tan B

[TOPIC 2] Trigonometric Identities

Summary

Trigonometric Identities

An equation involving trigonometric ratios of an angle is called a trigonometric identity, if it is true for all values of the angle(s) involved.

Following are the three trigonometric identities which are used to solve the basic trigonometric equations.

(i) $\sin^2\theta + \cos^2\theta = 1$;

(ii) $1 + \tan^2\theta = \sec^2\theta$;

(iii) $1 + \cot^2\theta = \text{cose}^2\theta$.

PREVIOUS YEARS' EXAMINATION QUESTIONS

TOPIC 2

1 Mark Questions

1. $4\tan^2 A - 4\sec^2 A$ is equal to:

(a) –1 (b) –4

(c) 0 (d) 4

[TERM 2, 2011]

2. $[(\sec A + \tan A)(1 - \sin A)]$ on simplification gives

(a) $\tan^2 A$ (b) $\sec^2 A$

(c) $\cos A$ (d) $\sin A$

[TERM 2, 2011]

3. Evaluate: $\sin^2 A + \cos^2 A + \cot^2 A$

[TERM 2, 2015]

4. Find the value of $(\sec^2\theta - 1) \cdot \cot^2\theta$

[TERM 2, 2016]

2 Marks Questions

5. If $\sqrt{3}\tan\theta = 3\sin\theta$, find the value of $\sin^2\theta - \cos^2\theta$.

[TERM 2, 2011]

6. If $\sin\theta - \cos\theta = \frac{1}{2}$, then find the value of $\sin\theta + \cos\theta$.

[TERM 2, 2012]

7. $\left[\frac{1-\tan A}{1-\cot A}\right]^2 = \tan^2 A$; $\angle A$ is acute

[TERM 2, 2014]

8. Prove the following identity: $\frac{\sin^4\theta + \cos^4\theta}{1 - 2\sin^2\theta\cos^2\theta} = 1$

[TERM 2, 2015]

3 Marks Questions

9. Prove that $(\mathrm{cosec}\,\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$.

[TERM 2, 2011]

10. Prove that: $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

[TERM 2, 2012]

11. Prove that: $\frac{(1+\tan^2 A)\cot A}{\mathrm{cosec}^2 A} = \tan A$

[TERM 2, 2012]

12. If $\tan\theta + \frac{1}{\tan\theta} = \sqrt{2}$, find the value of $\tan^2\theta + \frac{1}{\tan^2\theta}$

[TERM 2, 2013]

13. Prove that:

$$\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{2}{2\sin^2\theta - 1}$$

[TERM 2, 2013]

Summary

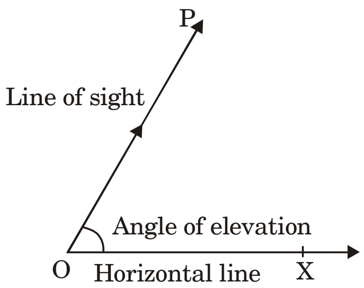
Introduction

LINE OF SIGHT

When an observer looks from a point O at an object P then the line OP is called the line of sight.

ANGLE OF ELEVATION

Assume that from a point O, we look up at an object P, placed above the level of our eye. Then, the angle which the line of sight makes with the horizontal line through O is called the angle of elevation of P, as seen from O.

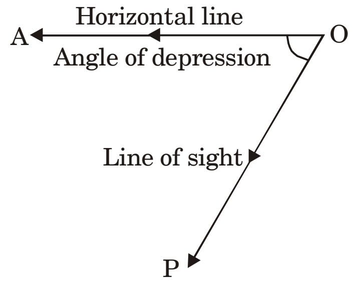


Example: Let OX be a horizontal line on the level ground and let a person at O be looking up towards an object P, say an aeroplane or the top of a tree or the top of a tower, or a flag at the top of a house.

Then, $\angle XOP$ is the angle of elevation of P from O.

ANGLE OF DEPRESSION

Assume that from a point O, we look down at an object P, placed below the level of our eye.



Then, the angle which the line of sight makes with the horizontal line through O is called the angle of depression of P, as seen from O.

PREVIOUS YEARS'

EXAMINATION QUESTIONS

1 Mark Questions

1. The angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of the tower is 45°. The height of the tower (in metres) is

(*a*) 15

(*b*) 30

(*c*) $30\sqrt{3}$

(*d*) $10\sqrt{3}$

[TERM 2, 2011]

2. A kite is flying at a height of 30 m from the ground. The length of string from the kite to the ground is 60 m. Assuming that there is no slack in the string, the angle of elevation of the kite at the ground is

(*a*) 45° (*b*) 30°

(c) 60° (*d*) 90°

[TERM 2, 2012]

3. The angle of depression of a car, standing on the ground, from the top of a 75 m high tower, is 30°. The distance of the car from the base of the tower (in m.) is:

(*a*) $25\sqrt{3}$ (*b*) $50\sqrt{3}$

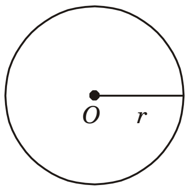
(*c*) $75\sqrt{3}$ (d) 150

[TERM 2, 2013]

Summary

Terms Related to Circles

CIRCLE



The set of points which are at a constant distance of units from a fixed point O is called a circle with centre O and radius = r units. The circle is denoted by $C(O, r)$.

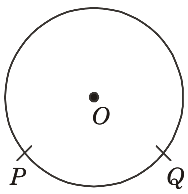
The fixed point O is called the centre and the constant distance r units is called its radius.

CIRCUMFERENCE

The perimeter (or length of boundary) of a circle is called its circumference.

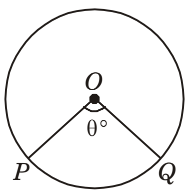
ARC

A continuous piece of a circle is called an arc of the circle.



In the given figure, PQ is an arc of a circle, with centre O, denoted by $\widehat{PQ}$. The remaining part of the circle, shown by the dotted lines, represents $\widehat{QP}$.

CENTRAL ANGLE

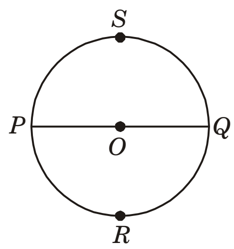


An angle subtended by an arc at the centre of a circle is called its central angle.

In the given figure of a circle with centre O, central angle of $\widehat{PQ} = \angle POQ = \theta^{\circ}$.

If $\theta^{\circ} < 180^{\circ}$ then the arc $\widehat{PQ}$ is called the minor arc and the arc $\widehat{QP}$ is called the major arc.

1.5 SEMICIRCLE

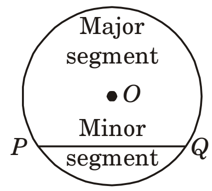


A diameter divides a circle into two equal arcs. Each of these two arcs is called a semicircle.

In the given figure of a circle with centre O, $\widehat{PRQ}$ and $\widehat{QSP}$ are semicircles.

An arc whose length is less than the arc of a semicircle is called a **minor arc.** An arc whose length is more than the arc of a semicircle is called a **major arc**.

SEGMENT

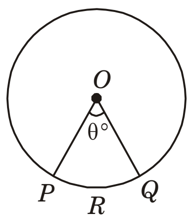


A segment of a circle is the region bounded by an arc and a chord, including the arc and the chord.

The segment containing the minor arc is called a minor segment, while the segment containing the major arc is the major segment.

The centre of the circle lies in the major segment.

SECTOR OF A CIRCLE



The region enclosed by an arc of a circle and its two bounding radii is called a sector of the circle.

In the given figure, $OPRQO$ is a sector of the circle with centre O.

If arc PQ is a minor arc then $OPRQO$ is called the minor sector of the circle.

The remaining part of the circle is called the major sector of the circle.

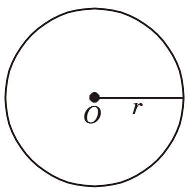
QUADRANT

One-fourth of a circular disc is called a quadrant. The central angle of a quadrant is 90º.

FORMULAE

Circumference and area of a circle

For a circle of radius r, we have



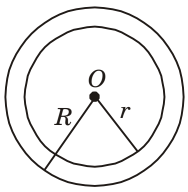
(i) Circumference of the circle = $2\pi r$

(ii) Area of the circle = πr^2

(iii) Area of the semicircle = $\frac{1}{2}\pi r^2$

(iv) Perimeter of the semicircle = $(\pi r + 2r)$

Area of a ring



Let R and r be the outer and inner radii of a ring.

Then, area of the ring = $\pi(R^2 - r^2)$.

Rotating wheels

(i) Distance moved by a wheel in 1 rotation = circumference of the wheel

(ii) Number of rotations in 1 minute

$$= \frac{\text{distance moved in 1 minute}}{\text{circumference}}$$

Rotation of the hands of a clock

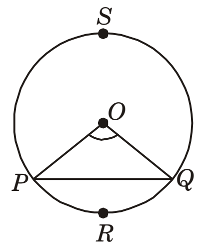
(i) Angle described by the minute hand of a clock in 60 minutes = 360°.

(ii) Angle described by the hour hand of a clock in 12hours = 360°.

Area of Sector and Segment of a Circle

Length of arc, area of sector and segment

Let an arc AB make an angle $\theta° < 180°$ at the centre of a circle of radius r. Then, we have



(i) Length of the arc $\widehat{PQ} = \frac{2\pi r\theta}{360} = \ell$

(ii) (a) Area of the sector $OPRQO$

$$= \frac{\pi r^2\theta}{360}$$

$$= \left(\frac{1}{2} \times \frac{2\pi r\theta}{360} \times r\right) = \left(\frac{1}{2} \times l \times r\right)$$

(b) Perimeter of the sector $OPRQO$

$$= OP + OQ + \text{length of arc } \widehat{PRQ} = \left(2r + \frac{2\pi r\theta}{360}\right)$$

(iii)(a) Area of the minor segment $PRQP$

= (area of the sector $OPRQO$) – (area of ΔOPQ)

$$= \left(\frac{\pi r^2\theta}{360} - \frac{1}{2}r^2 \sin\theta\right)$$

(b) Area of the major segment $QSPQ$

= (area of the circle)

– (area of the minor segment $PRQP$)

[TOPIC 1] Surface Area & Volume of a Solid

Summary

Surface Area and Volume of Solids

CUBOIDS AND CUBES

Cuboid : A cuboid is a solid figure, held by six rectangular plane regions

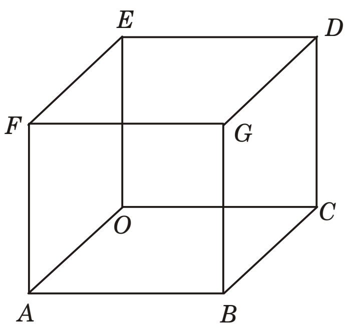
Here, In cuboid we have six faces namely

$AFGB, BGDC, GFED, OCDE, OEFA, OABC.$

We also have 12 edges, where two sides meet namely

$OA, AB, BC, OC, FG, EF, ED, OG, AF, OE, BG, CD.$

Cube: A cuboid in which all length, breadth, height are of equal lengths, is called a cube.



It also has six faces and twelve edges.

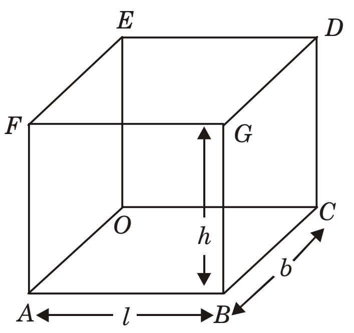
SURFACE AREA OF A CUBOID AND A CUBE

Total surface area of cuboid

$$= Ar(ABCO) + Ar(EFGD) + Ar(AOEF) + Ar(BCDG) + Ar(ABGF) + Ar(OCDE)$$

$$= \ell b + \ell b + bh + bh + \ell h + \ell h$$

$$= 2(\ell b + bh + \ell h)$$



Lateral surface area of cuboid

$$= 2(bh + \ell h)$$

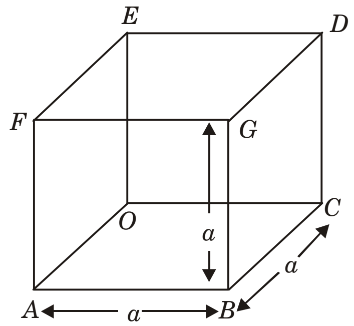
Where ℓ = length of cuboid

b = breadth of cuboid

h = height of cuboid

Total surface area of cube

Since cube is a cuboid in which length (ℓ) = breadth (b) = height (h) side of cube (a) i.e. $\ell = b = h = a$



$\Rightarrow$ Total surface area of cube

$$= 2(a \times a + a \times a + a \times a)$$

$$= 2(a^2 + a^2 + a^2)$$

$$\text{Area} = 2(3a^2) = 6a^2$$

Lateral surface area of cube

$$\text{Area} = 2(a \times a + a \times a)$$

$$= 2(a^2 + a^2) = 2 \times 2a^2$$

$$= 4a^2$$

So, lateral surface area of cube = $4a^2$

Where a = length of a side.

Length of diagonal of a cuboid

Length of diagonal $= OG = AD = BE = CF$

$$= \sqrt{\ell^2 + b^2 + h^2}$$

ℓ = length; b = breadth; h = height

Length of diagonal of a cube

Length of diagonal $= OG = AD = BE = CF$

$$= \sqrt{a^2 + a^2 + a^2}$$

$$= \sqrt{3a^2} = \sqrt{3}a \text{ unit}$$

Where a = length of a side.

Surface Area of a Right Circular Cylinder

CURVED SURFACE AREA OF A CYLINDER

$$= 2\pi rh$$

Where r = radius of base

h = height of cylinder.

TOTAL SURFACE AREA OF A CYLINDER

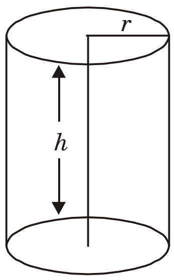
$$= 2\pi rh + 2\pi r^2$$

$$= 2\pi r(h + r)$$

r = radius of base

h = height of cylinder

$\pi = \dfrac{22}{7}$ or 3.14 approx.



VOLUME OF CYLINDER

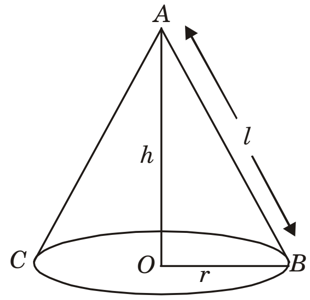
$$V = \pi r^2 h$$

Where r = radius of base

h = height of cylinder

Surface Area of a Right Circular Cone

CURVED SURFACE AREA OF A CONE



$C = \pi r\ell$

C = curved surface area

r = radius of base of cone

ℓ = slant height

$\ell = \sqrt{h^2 + r^2}$

TOTAL SURFACE AREA OF A CONE

$$T = \pi r\ell + \pi r^2 = \pi r(r + \ell)$$

Here, T = total surface area

r = radius of base of cone

ℓ = slant height of cone

VOLUME OF RIGHT CIRCULAR CONE

$$V = \frac{1}{3}\pi r^2 h$$

Where V = volume of cone

r = radius of base of cone

h = height of cone

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. A sphere of diameter 18 cm is dropped into a cylindrical vessel of diameter 36 cm, partly filled with water. If the sphere is completely submerged, then the water level rises (in cm) by

(*a*) 3

(*b*) 4

(*c*) 5

(*d*) 6

[TERM 2, 2011]

2. Volume and surface area of a solid hemisphere are numerically equal.

What is the diameter of hemisphere?

[TERM 2, 2017]

2 Marks Question

3. If the total surface area of a solid hemisphere is 462 cm^2, find its volume. [Take $\pi = \dfrac{22}{7}$]

[TERM 2, 2014]

3 Marks Questions

4. A hemispherical bowl of internal radius 9 cm is full of water. Its contents are emptied in a cylindrical vessel of internal radius 6 cm. Find the height of water in the cylindrical vessel.

[TERM 2, 2012]

[TOPIC 2] Conversion of Solid

Summary

Conversion of Solid from One Shape to Another

For commercial works and for industrial development work, we need to convert a solid into another solid of different shape or more than one solid of similar shape but with reduced size

A cylinder, a cone and a hemisphere are of equal base and have the same height. The ratio of their volume is 3:1:2

PREVIOUS YEARS'

EXAMINATION QUESTION

TOPIC 2

1 Mark Questions

1. The number of solid spheres, each of diameter 6 cm that can be made by melting a solid metal cylinder of height 45 cm and diameter 4cm, is:

(*a*) 3 (*b*) 5

(*c*) 4 (d) 6

[TERM 2, 2014]

2 Marks Questions

2. Two cubes, each of side 4 cm are joined end to end. Find the surface area of the resulting cuboid.

[TERM 2, 2011]

3. A solid sphere of radius 10.5 cm is melted and recast into smaller solid cones, each of radius 3.5 cm and height 3 cm. find the number of cones so formed. (Use $\pi = \frac{22}{7}$)

[TERM 2, 2012]

3 Marks Questions

4. Water in a canal, 6 m wide and 1.5 m deep, is flowing at a speed of 4 km/h. How much area will it irrigate in 10 minutes, if 8 cm of standing water is needed for irrigation?

[TERM 2, 2014]

5. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank which is 10 m in diameter and 2 m deep. If the water flows through the pipe at the rate of 4 km per hour, in how much time will the tank be filled completely?

[TERM 2, 2014]

6. A hemispherical bowl of internal diameter 36 cm contains liquid. This liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find the height of the each bottle, if 10% liquid is wasted in this transfer.

[TERM 2, 2015]

7. 504 cones, each of diameter 3.5 cm and height 3 cm, are melted and recast into a metallic sphere. Find the diameter of the sphere and hence find its surface area. [Use $= \frac{22}{7}$]

[TERM 2, 2015]

8. A well of diameter 4 m is dug 21 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width to form an embankment. Find the height of the embankment.

[TERM 2, 2016]

9. Water in a canal, 5.4 m wide and 1.8 m deep, is flowing with a speed of 25 km/hour. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation?

[TERM 2, 2016]

10. The dimensions of a solid iron cuboid are 4.4 m × 2. 6 m × 1.0 m. It is melted and recast into a hollow cylindrical pipe of 30 cm inner radius and thickness 5 cm. Find the length of the pipe.

[TERM 2, 2017]

[TOPIC 3] Frustum of a Right Circular Cone

Summary

Frustum of a Right Cicular Cone

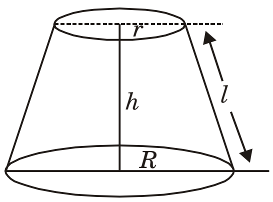
When a cone is cut by a plane parallel to the base of the cone then the portion between the plane and the base is called the frustum of the cone.

Let R and r be the radii of the base and the top of the frustum of a cone.

Let h be its height and ℓ be its slant height.

Then,

VOLUME OF THE FRUSTUM OF THE CONE



$$= \frac{\pi h}{3}[R^2 + r^2 + Rr] \text{ cubic units.}$$

LATERAL SURFACE AREA OF THE FRUSTUM OF THE CONE

$$= \pi\ell(R + r), \text{ where } \ell^2 = h^2 + (R - r)^2 \text{ sq units.}$$

TOTAL SURFACE AREA OF THE FRUSTUM OF THE CONE

= (area of the base) + (area of the top) + lateral surface area)

$$= [\pi R^2 + \pi r^2 + \pi\ell(R + r)]$$

$$= \pi[R^2 + r^2 + \ell(R + r)] \text{ sq units.}$$

PREVIOUS YEARS' EXAMINATION QUESTIONS TOPIC 3

1 Mark Question

1. A solid right circular cone is cut into two parts at the middle of its height by a plane parallel to its base. The ratio of the volume of the smaller cone to the whole cone is

   (*a*) 1 : 2

   (*b*) 1 : 4

   (c) 1 : 6

   (*d*) 1 : 8

[TERM 2, 2012]

3 Marks Questions

2. The radii of the circular ends of a bucket of height 15 cm are 14 cm and r cm (r<14 cm). If the volume of bucket is 5390 cm^3, then find the value of r. [Use $\pi = \frac{22}{7}$]

[TERM 2, 2011]

3. A solid metallic right circular cone 20 cm high and whose vertical angle is 60°, is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{12}$ cm, find the length of the wire.

[TERM 2, 2014]

[TOPIC 1] Mean, Median and Mode

Summary

It is generally observed that observations or data on a variable tend to gather around some central value. This gathering of data towards a central value is called **central tendency** or the middle value of the distribution, also known as middle of the data set.

A certain value representative of the whole data and signifying its characteristics is called an **average** of the data.

Three types of averages are useful for analyzing data.

They are : (i) Mean, (ii) Median, (iii) Mode.

Mean for a Grouped Frequency Dis Tribution

DIRECT METHOD

Step 1: For each class, find the class mark x_i, as

$$x_i = \frac{1}{2}\text{ (lower limit + upper limit)}$$

Step 2: Calculate $f_i x_i$ for each i.

Step 3: Use the formula : $= \dfrac{\sum(f_i x_i)}{\sum f_i}$.

Assumed-Mean Method

Following steps are taken to solve cases by assumed-mean method.

Step 1: For each class interval, calculate the class mark x_i by using the

formula: $x_i = \dfrac{1}{2}$(lower limit + upper limit).

Step 2: Choose a value of x_i in the middle as the assumed mean and denote it by A.

Step 3: Calculate the deviations $d_i = (X_i - A)$ for each i.

Step 4: Calculate the $(f_i d_i)$ for each i.

Step 5: Find $n = \sum f_i$.

Step 6: Calculate the mean, $\bar{x}$, by using the formula:

$$\bar{x} = A + \frac{\sum f_i d_i}{n}.$$

Step-Deviation Method

Following steps are taken to solve cases by step-deviation method.

Step 1: For each class interval, calculate the class mark x_i by using the

formula: $x_i = \dfrac{1}{2}$(lower limit + upper limit).

Step 2: Choose a value of x_i in the middle of the x_i column as the assumed mean and denote it by A.

Step 3: Calculate h = [(upper limit) – (lower limit)].

Step 4: Calculate $u_i = \dfrac{(x_i - A)}{h}$ for each class.

Step 5: Calculate $f_i u_i$ for each class and find $\sum(f_i u_i)$.

Step 6: Calculate the mean, by using the formula:

$$\bar{x} = A + \left[h\frac{\sum(f_i u_i)}{\sum f_i}\right].$$

Mode

It is that value of a variate which occurs most often. More precisely, mode is that value of the variable at which the concentration of the data is maximum.

Modal Class : In a frequency distribution, the class having maximum frequency is called the modal class.

Formula for Calculating Mode:

We have:

Mode, $M_0 = \ell + h.\left[\dfrac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)}\right]$, where

ℓ = lower limit of the modal class interval;

f_1 = frequency of the modal class;

f_0 = frequency of the class preceding the modal class;

f_2 = frequency of the class succeeding the modal class;

h = width of the class interval.

Method for Finding the Median for Grouped Data

Median of a distribution is the value of the variable which divides it into two equal parts. It is the value which exceeds and is exceeded by the same number of observation i.e., it is the value such that the number of observation above it is equal to the number of observation below it.

In case of grouped frequency distribution, the class corresponding to the cumulative (c.f) just greater than $\frac{N}{2}$ is called the median class.

Following steps are involved in finding the median of the given frequency distribution.

Step 1: For the given frequency distribution, prepare the cumulative frequency table and obtain $N = \Sigma f_i$.

Step 2: Find $(N/2)$.

Step 3: Find the cumulative frequency just greater than $(N/2)$ and find the corresponding class, known as median class.

Step 4: Use the formula:

Median, $Me = \ell + \left[h \times \frac{\left(\frac{N}{2} - c\right)}{f}\right]$, where

ℓ = lower limit of median class,

h = width of median class,

f = frequency of median class,

c = cumulative frequency of the class preceding the median class, $N = \Sigma f_i$.

Relationship Among Mean, Median and Mode

We have, Mode = 3(Median) –2(Mean)

or

Median = Mode $+\frac{2}{3}$ (Mean – Mode)

or

Mean = Mode $+\frac{3}{2}$ (Median – Mode)

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. If the mode of some data is 7 and their mean is also 7, then their median is:
   (*a*) 10 (*b*) 9
   (*c*) 8 (*d*) 7 **[TERM 1, 2011]**
2. Relationship among mean, median and mode is:
   (*a*) 3 Median = Mode + 2 Mean
   (*b*) 3 Mean = Median + 2 Mode
   (*c*) 3 Mode = Mean = 2 Median
   (*d*) Mode = 3 Mean – 2 Median **[TERM 1, 2012]**
3. Monthly pocket money of 50 students of a class are given in the following distribution:

Monthly pocket money (in Rs)	0 – 50	50 – 100	100 – 150	150 – 200	200 – 250	250 – 300
Number of students	2	7	8	30	12	1

Find modal class and also give class rank of the modal class. **[TERM 1, 2014]**

[TOPIC 2] Cumulative Frequency Distribution

Summary

Graphical Representation of Cumulative Fequency Distribution

Let a grouped frequency distribution be given to us.

FOR A 'LESS THAN' SERIES

On a graph paper, mark the upper class limits along the x-axis and the corresponding cumulative frequencies along the y-axis.

(i) On joining these points successively by line segments, we get a polygon, called cumulative frequency polygon.

(ii) On joining these points successively by smooth curves, we get a curve, known as cumulative frequency curve or an ogive.

- ***Taken a point $A\left(0, \frac{N}{2}\right)$ on the y-axis and draw AP II x-axis, cutting the above curve at a point P.***

  Draw PM ⊥ x-axis, cutting the x-axis at M. Then, median = length of OM.

FOR A 'GREATER THAN' SERIES

On a graph paper, mark the lower class limits along the x-axis and the corresponding cumulative frequencies along the y-axis.

(i) On joining these points successively by line segments, we get a polygon, called cumulative frequency polygon.

(ii) On joining these points successively by smooth curves, we get a curve, known as cumulative frequency curve or an ogive.

- ***Let P be the point of intersection of 'less than' and 'more than' curves. Draw PM ⊥ x-axis, cutting x-axis at M. Then, median = length of OM.***

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 2

1 Mark Questions

1. The following are the ages of 300 patients getting medical treatment in a hospital on a particular day. Write the above distribution as less than type cumulative frequency distribution.

Age (in years)	10-20	20-30	30-40	40-50	50-60	60-70
Cumulative frequency	60	42	55	70	53	20

[TERM 1, 2011]

Summary

Probability

Probability is a concept which numerically measures the degree of certainty of the occurrence of events.

EXPERIMENT

An operation which can produce some well-defined outcomes is called an experiment.

I. Tossing a coin. When we throw a coin, either a head (H) or a tail (T) appears on the upper face.

II. Throwing a die. A die is a solid cube, having 6 faces, marked 1, 2, 3, 4, 5 and 6, or having 1, 2, 3, 4, 5 and 6 dots.

III. A deck of playing cards has in all 52 cards.

(i) It has 13 cards each of four suits, namely Spades, clubs, hearts and diamonds.

(a) Cards of spades and clubs are black cards.

(b) Cards of hearts and diamonds are red cards.

(ii) Kings, queens and jacks are known as face cards.

EVENT

The collection of all or some of the possible outcomes is called an event.

Examples:

(i) In throwing a coin, H is the event of getting a head.

(ii) Suppose we throw two coins simultaneously and let E be the event of getting at least one head. Then, E contains HT, TH, HH.

EQUALLY LIKELY EVENTS

A given number of events are said to be equally likely if none of them is expected to occur in preference to the others.

Probability of Occurrence of an Event

Probability of occurrence of an event E, denoted by $P(E)$ is defined as:

$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Total number of possible outcomes}}$$

COMPLEMENTARY EVENT

Let E be an event and (not E) be an event which occurs only when E does not occur.

The event (not E) is called the complementary event of E.

Clearly, $P(E) + P(\text{not } E) = 1$.

$\therefore\ P(E) = 1 - P(\text{not } E)$.

Some Special Sample Spaces

A die is thrown once: $S = \{1, 2, 3, 4, 5, 6\}; n(S) = 6$

A coin is tossed once: $S = \{H, T\}; n(S) = 2$

A coin is tossed twice or Two coins are tossed simultaneously: $S = \{HH, HT, TH, TT\}; n(S) = 4 = 2^2$

A coin is tossed three times or Three coins are tossed simultaneously:

$$S = \begin{Bmatrix} HHH, HHT, HTH, THH \\ TTT, TTH, THT, HTT \end{Bmatrix}; n(S) = 8 = 2^3$$

Two dice are thrown together or A die is thrown twice:

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

$n(S) = 6^2$

- *1.* $P(E) = \dfrac{\textit{number of favourable cases}}{\textit{total number of cases}}$

  2. $P(E) + P(\text{not } E) = 1$

  3. $0 \le P(E) \le 1$

  4. ***Sum of the probabilities of all the outcomes of random experiment is 1.***